Mini-courses

Three sides to the circle
Danny Calegari, University of Chicago.

The circle, its symmetries, and its connection to low-dimensional geometry, topology and dynamics has been a major theme in Etienne's work for decades. In this minicourse we discuss three interesting examples that illustrate some of the richness of the theory; explicitly, we discuss the theories of Ziggurats; of universal circles; and of quasigeodesic flows.

Schedule: Monday 11:30, Tuesday 10:30 and Wednesday 9:00.

Lattices and Invariant Random Subgroups
Tsachik Gelander, Weizmann Institute.

An Invariant Random Subgroup (IRS) in a locally compact group G is a conjugacy invariant probability measure on the space of closed subgroups of G. Every lattice is associated with an IRS. The (compact) G-space $\mathrm{IRS}(G)$ of all such measures is a useful tool to analyse its special (lattices) points, mainly in order to study their asymptotic properties. In the other direction, various results from the theory of lattices can be extended to to the framework of general IRS. In the mini course, I will elaborate on these approaches in the case where G is an algebraic group over a local field.

Schedule: Tuesday 11:40, Wednesday 10:30 and Thursday 9:00.

Random walks on hyperbolic groups
Sébastien Gouëzel, CNRS-Université Rennes 1.

The behaviour of random walks on groups is most often related to the geometry of the groups. In particular, it is expected that random walks on hyperbolic groups look a lot like very well understood random walks, such as random walks on rank 1 simple Lie groups or on hyperbolic spaces. In these cases, using representation theory, one can show that the local limit problem (i.e., the asymptotics of the probability to return to a fixed neighborhood of the origin) involves a universal exponent $3/2$. We will explain why this remains true in hyperbolic groups, using geometric arguments.

Schedule: Tuesday 9:00, Thursday 10:30 and Friday 9:00.

Monday

10:15 **Continuity of Lyapunov exponents for random matrix products**
Artur Avila, CNRS-Université Paris 7-IMPA.

Let $A_1, ..., A_n \in GL(d)$ and let $p_1, ..., p_n > 0$ be such that $\sum p_i = 1$. We consider the random product of the A_i, taken independently with probability p_i, concentrating on the Lyapunov exponents $\lambda_1 \geq ... \geq \lambda_d$. We prove that the λ_j depend continuously on the matrices A_i and the weights p_i. This is joint work with Alex Eskin and Marcelo Viana.

11:30 **Three sides to the circle, part 1**
Danny Calegari, University of Chicago.

14:30 **The moduli space of closed anti-de Sitter 3-manifolds**
Nicolas Tholozan, University of Luxembourg.

It is well known that closed hyperbolic surfaces of fixed genus g are parametrized up to isometry by a complex manifold of dimension $3g-3$ whose topology is rather well understood. In this talk, I will describe the analog of this moduli space for closed *anti de-Sitter* 3-manifolds, that is, closed 3-manifolds endowed with a Lorentz metric of constant curvature -1. Our description relies on the work of Kulkarni–Raymond and Kassel relating those manifolds to representations of surface groups into $\mathrm{PSL}(2, \mathbb{R})$.

16:00 **The Furstenberg boundary and C*-simple groups**
Emmanuel Breuillard, CNRS-Université Paris-Sud.

A discrete group is called C*-simple if its reduced C*-algebra is simple. C*-simple groups have no non-trivial amenable normal subgroups, but whether this is a characterization of C*-simplicity remains an open problem. Recently Kalantar and Kennedy found a dynamical characterization of C*-simplicity: a discrete group is C*-simple if and only it admits a topologically free boundary action. I will describe this and further recent work with Kalantar, Kennedy and Ozawa, in which we investigate C*-simplicity and the unique trace property for a large class of discrete groups.

17:10 **Anti-de Sitter 3-manifolds, Margulis spacetimes, and their higher-dimensional analogues**
Fanny Kassel, CNRS-Université de Lille.

In 1983, Margulis constructed the first examples of proper affine actions of free groups on $\mathbb{R}^3$. I will describe the geometry and topology of the corresponding quotients, which are flat Lorentzian 3-manifolds known as Margulis spacetimes, and of their negatively-curved counterparts,

known as anti-de Sitter 3-manifolds. It is actually possible to perform similar constructions in higher dimension with a variety of discrete groups, not necessarily free. This is joint work with J. Danciger and F. Guéritaud.

Tuesday

9:00 **Random walks on hyperbolic groups, part 1**
Sébastien Gouëzel, CNRS-Université Rennes 1.

10:30 **Three sides to the circle, part 2**
Danny Calegari, University of Chicago.

11:40 **Lattices and Invariant Random Subgroups, part 1**
Tsachik Gelander, University of Jerusalem.

14:45 **Complex dynamics and elliptic curves**
Laura DeMarco, Northwestern University.

In this talk, I will explain some connections between recent research in dynamical systems and the classical theory of elliptic curves and rational points. I will begin with the theorem of Mordell and Weil from the 1920s, presented from a dynamical point of view. I plan to finish by describing a dynamical/geometric proof of a recent result of Masser and Zannier about torsion points on elliptic curves and "unlikely intersections".

16:15 **Mathematical Imagery**
Jos Leys.

Visualization in mathematics can help understanding and is thus a great didactic tool, but mathematical imagery can also be just beautiful. In this talk I show a number of cases such as Kleinian groups, 3D fractals, hyperbolic tilings and others, where both these aspects are combined.

17:30 **Remarks on laminations and foliations of compact spaces**
Dennis Sullivan, Stony Brook-CUNY.

There are very nice statements that are somewhat isolated and suggest problems and applications that might connect them to each other and to other areas.

- Every oriented one dimensional lamination is the full boundary of an oriented two dimensional lamination with boundary.
- A holomorphic lamination in the complex projective plane cannot have a non atomic transverse invariant measure.

– Every two dimensional lamination has a unique continuous metric with hyperbolic leaves in each leafwise conformal class or there must be a transverse invariant measure (or both).

– The inverse limit of a branched covering space of a holomorphic self mapping of the Riemann sphere admits an invariant subspace which is laminated by complex planes relative to a finer topology where every open set has positive measure for the lebesgue measure on leaves cross the maximal entropy measure (interpreted as a transverse invariant measure).

– For any diffeomorphism of a closed manifold with invariant measure of positive entropy there are two laminations outside the space with (expanding and contracting) dynamics and transverse invariant measures. These inject equivariantly into the smooth dynamics consistently with the partial hyperbolic structure so that their intersection has full measure.

Wednesday

9:00 **Three sides to the circle, part 3**
Danny Calegari, University of Chicago.

10:30 **Lattices and Invariant Random Subgroups, part 2**
Tsachik Gelander, University of Jerusalem.

11:40 **Markoff triples, graphs and strong approximation.**
Peter Sarnak, IAS Princeton.

We describe joint work with Bourgain and Gamburd concerning divisibility properties of Markoff numbers and the execution of an elementary sieve on these. These numbers arise as the coordinates of Markoff triples which are an orbit of a group of (nonlinear) affine morphisms of affine 3-space. While a theory of an affine sieve for linear actions has been developed, the non-linearity and sparsity in the above setting introduces serious obstacles.

Thursday

9:00 **Lattices and Invariant Random Subgroups, part 3**
Tsachik Gelander, University of Jerusalem.

10:30 **Random walks on hyperbolic groups, part 2**
Sébastien Gouëzel, CNRS-Université Rennes 1.

11:40 **Harmonic maps and heat flows on hyperbolic spaces**
Vladimir Markovic, Caltech.

We prove that any quasi-conformal map of the $(n-1)$-dimensional sphere when $n > 2$ can be extended to a smooth quasi-isometry F of the n-dimensional hyperbolic space such that the heat flow starting with F converges to a quasi-isometric harmonic map. This implies the Schoen-Li-Wang conjecture that every quasi-conformal map of the $(n-1)$-sphere can be extended to a harmonic quasi-isometry when $n > 2$ (very recently, we announced the corresponding result when $n = 2$ which was the original Schoen Conjecture, but this proof does not involve heat flows). The cornerstones of our theory of the heat flow convergence are the Hamilton parabolic maximum principles for sub-solutions of the heat equation, the diffusion of heat in the hyperbolic space (the heat diffuses "ballistically" in the hyperbolic space), and the Mostow rigidity. This is joint work with Marius Lemm.

14:45 **Diffeomorphisms and smooth mapping class groups of Cantor sets**
Sebastian Hurtado, Université Paris 6.

Mapping class groups of Cantor sets in surfaces appear naturally in the study of group actions on surfaces. J. Bavard recently showed some of these groups act faithfully in certain hyperbolic spaces. Funar-Neretin described the relation of some of these groups to certain Thompson's like groups. I'll talk about these groups, its dynamics and possible potential applications. I'll explain some results obtained in collaboration with E. Militon about the simplest Cantor set in the sphere and its smooth mapping class group.

16:15 **Chain reactions in all directions**
Tadashi Tokieda, Cambridge University.

17:30 **Hamiltonian diffeomorphisms and persistence modules**
Leonid Polterovich, Tel-Aviv University.

I'll discuss robust obstructions to representing a Hamiltonian diffeomorphism as a full power, with applications to geometry and dynamics. The approach is based on the theory of persistence modules. Joint work with Egor Shelukhin.

Friday

9:00 **Random walks on hyperbolic groups, part 3**
Sébastien Gouëzel, CNRS-Université Rennes 1.

10:30 **A nonamenable finitely presented group of piecewise projective homeomorphisms**
Yash Lodha, Cornell University.

I will describe a finitely presented group of piecewise projective homeomorphisms of the real line. This provides a new example of a finitely presented group which is nonamenable and yet does not contain a non-abelian free subgroup. The example is torsion free and of type F_∞. A portion of this is joint work with Justin Moore.

11:40 **On the group of real analytic diffeomorphisms**
Takashi Tsuboi, University of Tokyo.

We review a result by Herman which says that the identity component of the group of real analytic diffeomorphisms of the n-dimensional torus is a simple group. Then we discuss the perfectivity of the identity component of the group of real analytic diffeomorphisms of manifolds which admit nontrivial circle actions and explain several known results. If time allows, we also talk about other problems on the group of real analytic diffeomorphisms.