Juste un petit mot . . .

What follows is an excerpt of slides shown during my talk, on 2 July 2015, at *Geometries in Action*, the conference in honor of Etienne Ghys's 60th birthday (還暦祝).

However, the most important part of the talk consisted of

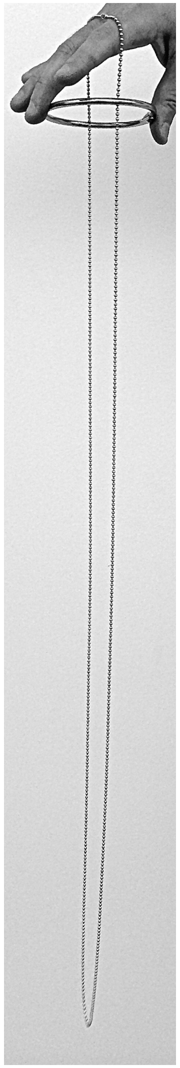
several table-top demos (actual experiments, not simulations) and

footages of a few experiments taken by high-speed cameras,

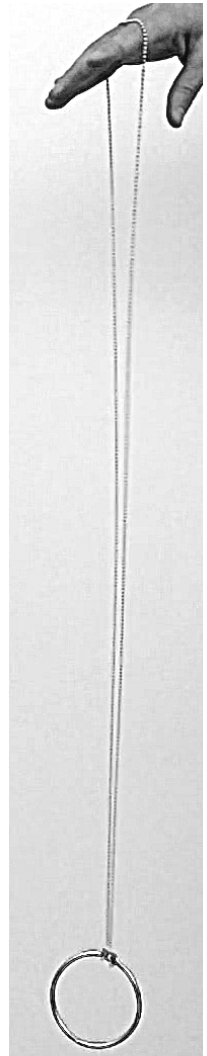
strewn throughout the talk. In addition, there were a magic trick plus a speech at the end. These could not be shown on the slides.

I cordially thank the organizers for allowing me to participate in this joyous occasion.

Chain



Reactions



to Etienne Ghys

from Tadashi Tokieda

Chain fountain [Biggins & Warner 2014]

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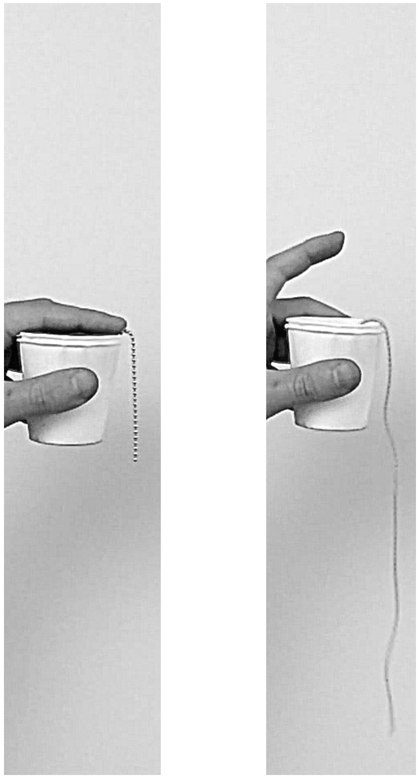
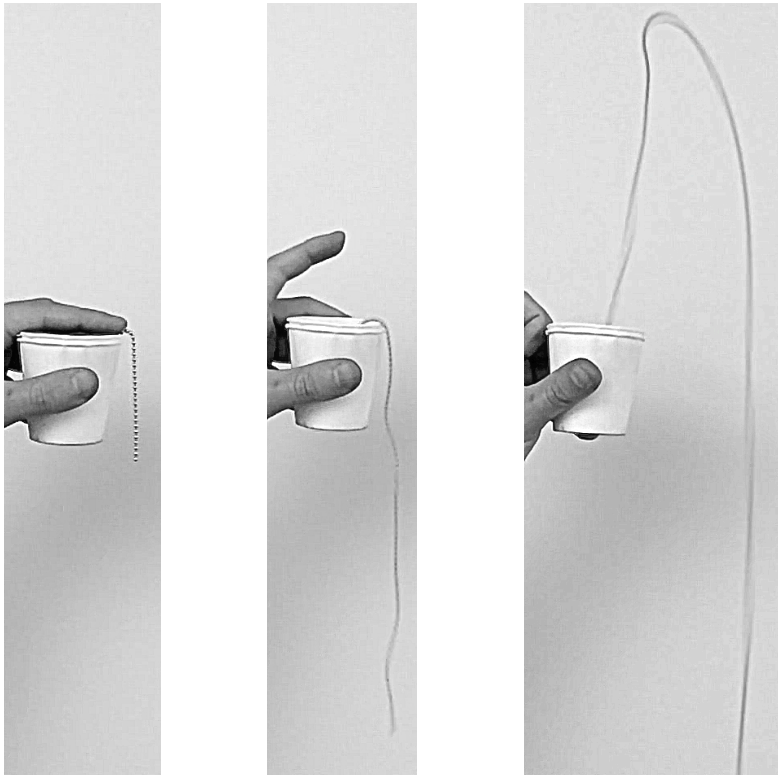


Chain fountain [Biggins & Warner 2014]



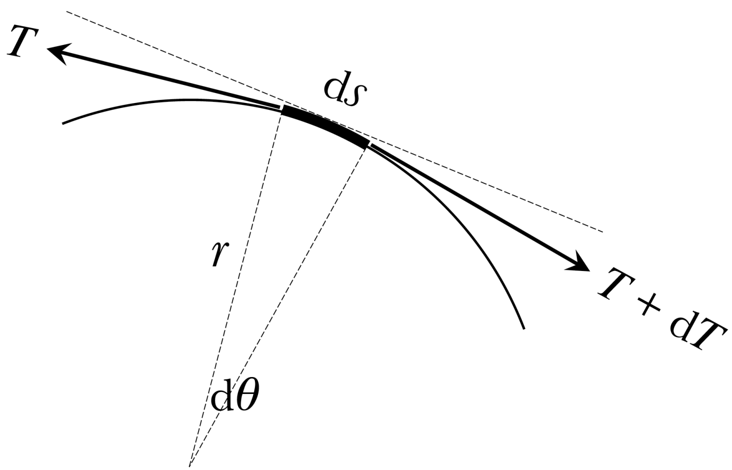
Q : What makes the chain stand up ?

It’s a *superstition* that a chain finds it harder to round a sharp corner

easier . . . gentle . . .

Theorem [19th century, forgotten] :

Once it gets going,
a chain can flow *in any shape* in neutral equilibrium.



Normal comp of tension

$$(T + \mathrm{d}T)\sin\frac{\mathrm{d}\theta}{2} + T\sin\frac{\mathrm{d}\theta}{2} \approx T\mathrm{d}\theta = \frac{T\mathrm{d}s}{r}$$

$$\Rightarrow \quad \rho\,\frac{v^2}{r} = \frac{T}{r} + n$$

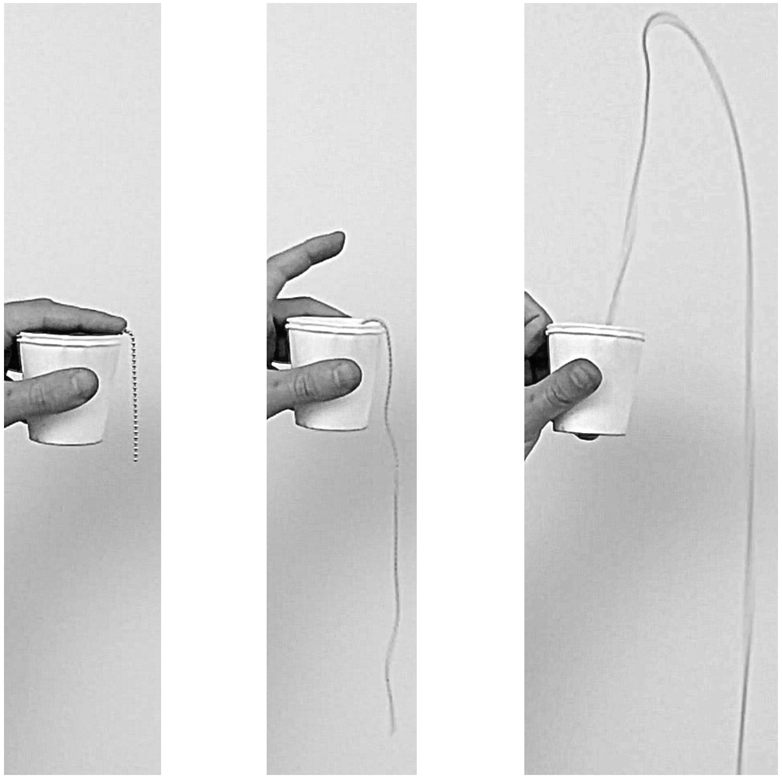
If flow fast enough $\quad v^2 \gg \dfrac{rn}{\rho} \quad$ then

$$\boxed{T \approx \rho v^2}$$

indep of
radius of curvature.

It's a *superstition* that a chain finds it harder to round a sharp corner
easier . . . gentle . . .

Chain fountain [Biggins & Warner 2014]



Q : What makes the chain stand up ?

A : ***Anomalous reaction***
due to singular bending stiffness.

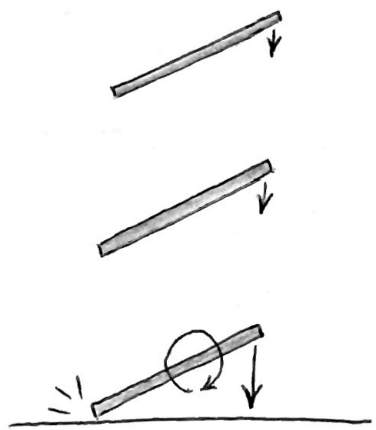
When an external stimulus acts on a system,
sometimes the system shows a reaction *not opposite to* the action :

anomalous reaction

in apparent violation of Newton's 3rd law.

Toy model

Free-falling rod . . .



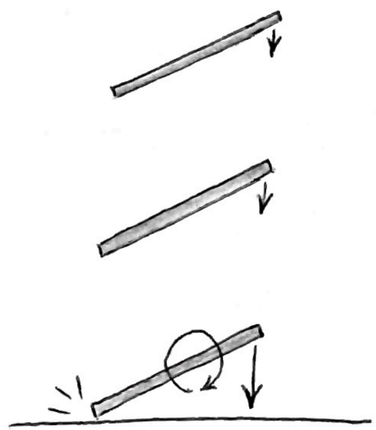
When one end impacts,
the rod flips,
the other end *accelerates downward* .

[Ruina’s movie]

Toy model

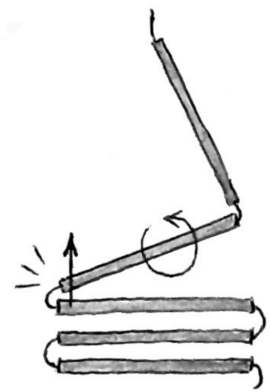
Imagine a chain of rods.

Free-falling rod . . .



When one end impacts,
the rod flips,
the other end *accelerates downward* .

As one end is picked up,
the rod flips,
the other end bangs
against the pile

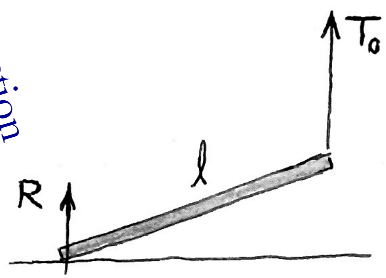


⇒ *upward kick*

. . . building up to a fountain.

The model hinges on the ***bending stiffness*** of the rod.

anomalous reaction



In time $\mathrm{d}t$ we yank to v a mass $\rho v \mathrm{d}t$ so the momentum is $\rho v^2 \mathrm{d}t$:

$$T_0 + R = \rho v^2$$

Force / mass $\rho v^2 / \rho \ell$ is acceleration,

divide by radius $\frac{l}{2}$ to get angular acceleration.

The torque is

$$\frac{l}{2} T_0 - \frac{l}{2} R = \frac{1}{12} \rho\, l^3 \times 2\, \frac{v^2}{l^2} = \frac{1}{6} \rho\, l v^2$$

Solution

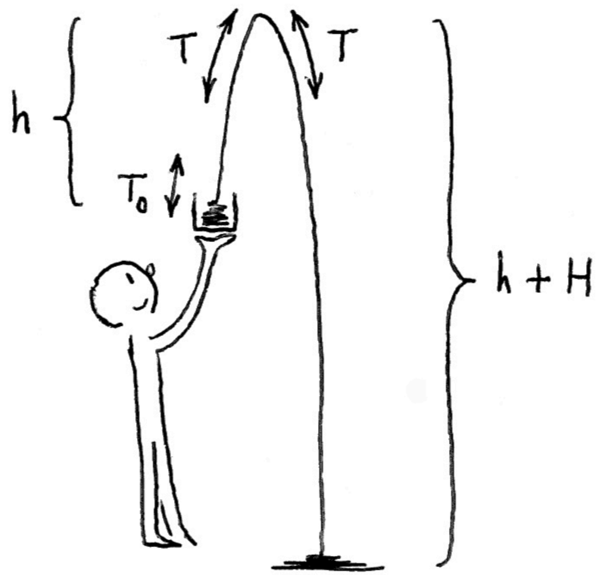
$$T_0 = \frac{2}{3} \rho v^2 , \quad R = \frac{1}{3} \rho v^2$$

indep of l so continuum limit $l \to 0$ valid.

(Note work by the force is $\rho v^2 \cdot v \mathrm{d}t$ whereas KE received is $\frac{1}{2} \rho v \mathrm{d}t \cdot v^2$

$\Rightarrow$ *half* of the energy gets lost to the ***shock*** dissipation.)

$$T_0 = \frac{2}{3}\,\rho v^2\,, \qquad R = \frac{1}{3}\,\rho v^2$$



rise at peak drop

$$T_0 + \rho h g \approx T \approx \rho(h+H)g$$

Also know

$$T \approx \rho v^2$$

$$\Rightarrow \quad \frac{h}{H} \approx \frac{1}{2}\,, \qquad \frac{v}{\sqrt{2gH}} \approx \frac{\sqrt{3}}{2}$$

The approximations in the model ok if flow fast enough $v^2 \gg \dfrac{rn}{\rho}$.

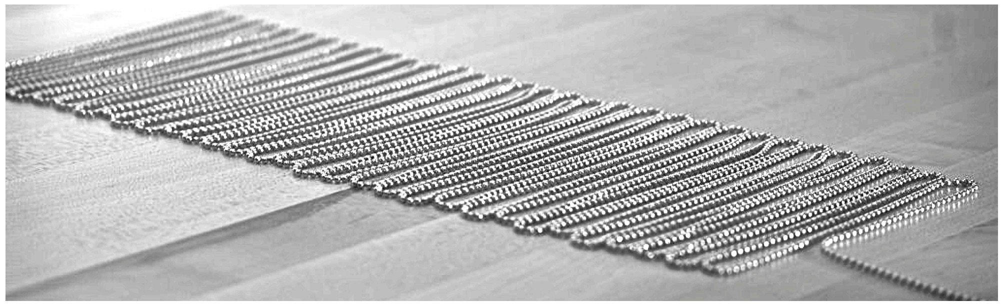
Here $n \sim \rho g$, so ok if $H \gg r$.

[Steinhardt’s movie, regular]

[Steinhardt's movie, variable]

Chain arch, regular [Hanna & Santangelo 2012]

Zigzag monolayer, drawn out —
arrangement and stimulus *horizontal*, no vertical comp of anything.



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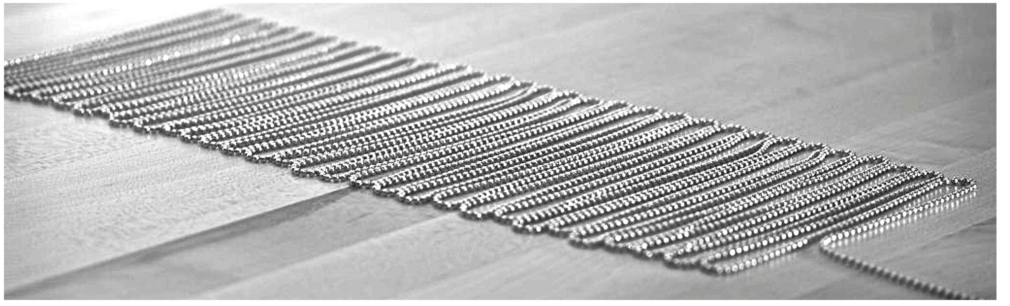
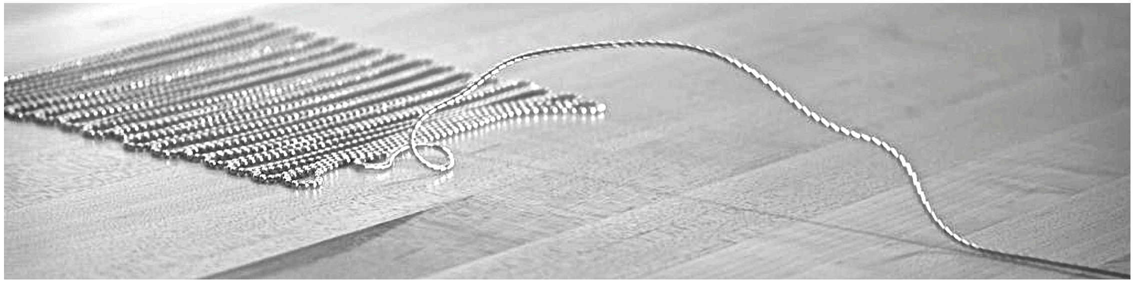


But the chain stands up *vertically* in an arch :



anomalous reaction
in a direction *indep* of the direction of stimulus.

Q : Why stand up ?

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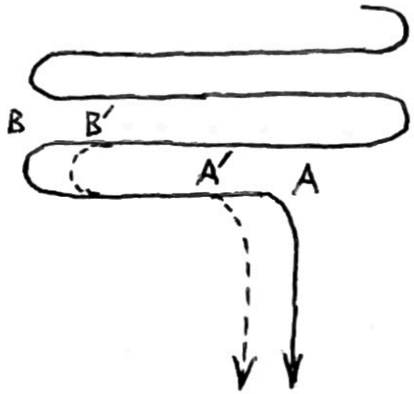
As the chain
is drawn out



it yanks
a local ‘rod’
in the bend

et voilà, a *curl* !
(with curvature
near singularity)

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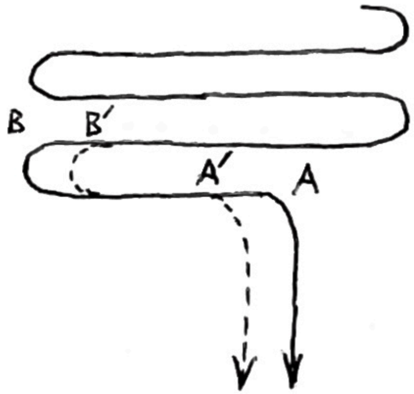

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And as the tail
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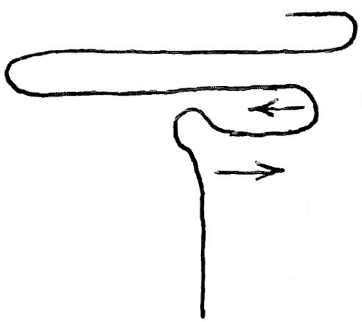

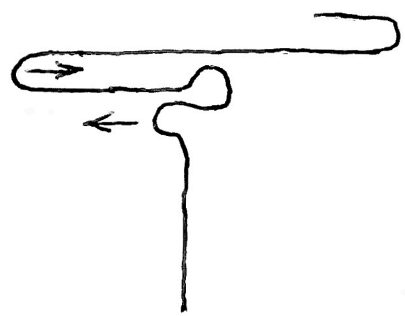

the curls pile up . . .

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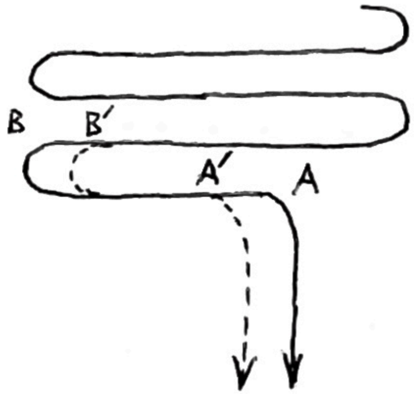

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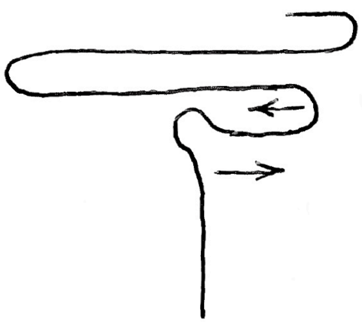

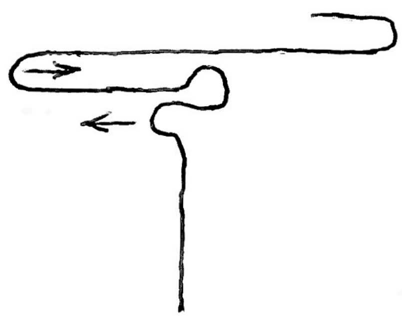

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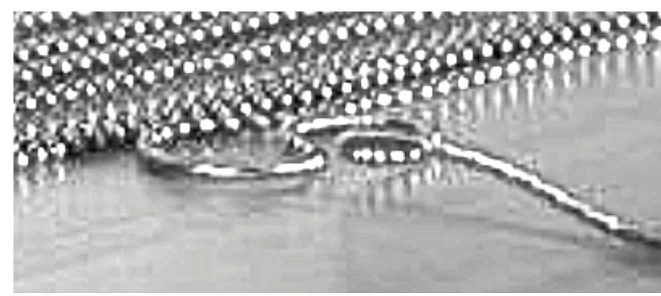
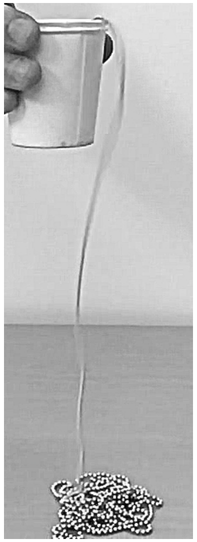


A : Accumulated curvatures
buckle off as torsion.

Chain arch, randomized [T^2 2014]

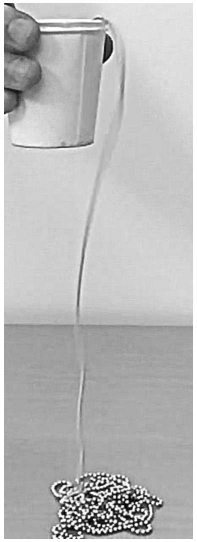


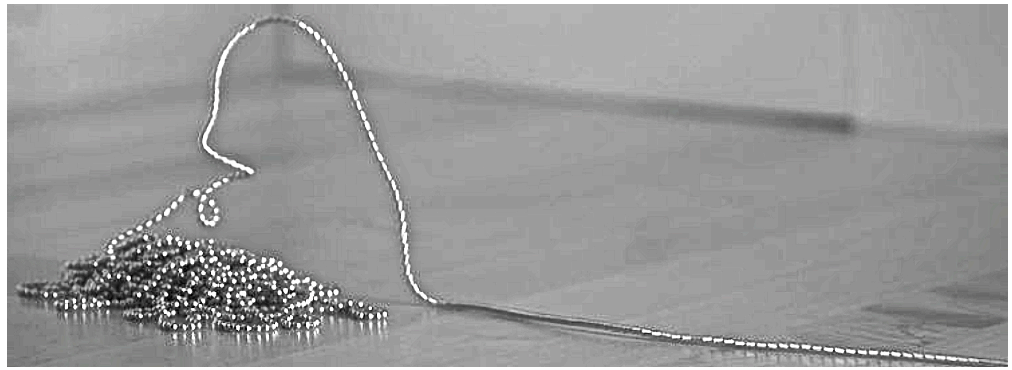
The phenomenon is *generic* : all we need are

- singular bending stiffness
- shock
- supply of 'critical geometry' as seeds

[Steinhardt’s movie, randomized]

Chain arch, randomized [T² 2014]





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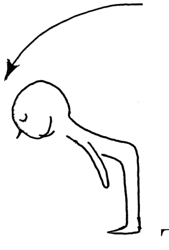
Conseq of $T \approx \rho v^2$:

v is the speed of material transport. But the wave speed is $\sqrt{\frac{T}{\rho}}$.

$\Rightarrow$ All waves on a flowing chain are *standing waves* , whence the stability of shape.

Many happy returns to Etienne



T^2

ENS Lyon, July 2015